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Optimal Modal Control

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ABSTRACT

A computationally simple sequential algorithm for designing an optimal modal controller is presented. The approach is capable of systematically calculating a set of appropriate weighting matrices for the cost functional in the quadratic performance index such that the resulting optimal control law would not only minimize the cost functional, but also would place the eigenspectrum of the closed loop system at some predefined locations.

1) INTRODUCTION

In applications of optimal Linear Quadratic Regulator (LQR) theory to multi-input systems there has not been a clear cut procedure for selecting the weighting matrices in order to achieve a set of desired closed loop eigenspectrum. Traditionally, to achieve some acceptable transient response behavior, these weights have been selected by trial and error, and are assumed to be diagonal to simplify this process. Recently the weight selection in the LQR problem have been considered by a number of authors [1-4]. The problem is basically that of calculating an appropriate state weighting matrix for a fixed choice of the control weighting matrix such that the resulting optimal control law would place the closed loop eigenspectrum of the system at some predefined locations. The approach used in [1-3] are based on the mirror image property of the symplectic Hamiltonian matrix. The approach presented in [1], and [2] is only capable of placing the real real part of the eigenspectrum, while the approach presented in [3] is capable of placing both the real and imaginary part of the closed loop poles. In [4] an approach based upon the entire eigenstructure assignment of Porter and D'Azzo [5] is presented.

In this paper optimal pole placement by weight selection is addressed using the entire eigenstructure assignment approach of [5]. However, the difference between this approach and that of [4] is that the present approach is a sequential one. That is, at every sequential stage either a real pole or a complex conjugate pair is placed, and an appropriate weighting matrix and the corresponding optimal control law to accomplish this is found. The advantage offered by this approach over that of [4] is its computational simplicity, especially for large scale systems whose only dominant or unstable modes need to be assigned. This simplicity results from the fact that at every stage of the sequential process, the

system is aggregated to a first or a second order system depending on whether a real pole or a complex conjugate pair is to be assigned. In this way all the computations required for eigenvalue, eigenvector assignment, and weight selection, along with calculation of the optimal control law involve scalar, or second order matrix computation regardless of the system's dimensionality. This feature of the proposed algorithm can reduce the computational effort drastically in particular when dealing with high order systems. The approach is capable of assigning all the eigenvalues to new locations, or assigning a subset of them to new locations, and retaining the remaining ones. This is another important feature of the algorithm, since no control effort will be used to move nondominant modes of the system.

2) Optimal Modal Controller

Consider a linear time invariant system described by

$$\dot{\underline{X}} = \underline{A}\,\underline{X} + \underline{B}\,\underline{U} \tag{1}$$

where $\underline{X} \in \mathbb{R}^n$, and $\underline{U} \in \mathbb{R}^m$ are the state, and input vectors respectively, and $\underline{A}$ and $\underline{B}$ are matrices of appropriate dimension. It is desired to find an optimal control law $\underline{U}^*$ such that the following quadratic cost functional is minimized

$$J = \frac{1}{2}\int_0^\infty \left(\|\underline{X}\|^2_{\underline{Q}} + \|\underline{U}\|^2_{\underline{R}} \right) dt \tag{2}$$

subject to (1). The weighting matrices $\underline{Q}$, and $\underline{R}$ are of appropriate dimension and it is required for $\underline{R}$ to be positive definite. However, it has been shown [2-4], that the weighting matrix $\underline{Q}$ need not satisfy the customary positive, or at least positive semidefinite requirement. In addition assume that the control inputs are unbounded, and the system is completely controllable. The problem stated above is the standard LQR

problem. In this paper, in addition to the above it is required to calculate a desired state weighting matrix $\underline{Q}$ such that the resulting optimal control law would assign the closed loop eigenspectrum to some predefined locations in the left hand plane.

The optimal control law for the standard LQR problem is given by [6]

$$\underline{U}^{*} = \underline{K}\,\underline{X} = -\underline{R}^{-1}\underline{B}^{T}\underline{P}\,\underline{X} \tag{3}$$

where $\underline{P}$ is the symmetric positive semidefinite solution of the algebraic matrix Riccati equation (AMRE), which is given by

$$\underline{P}\,\underline{A} + \underline{A}^{T}\underline{P} - \underline{P}\,\underline{B}\,\underline{R}^{-1}\underline{B}^{T}\underline{P} + \underline{Q} = 0 \tag{4}$$

Given the optimal control law in (3), it is well known [6] that the closed loop system is asymptotically stable and is given by

$$\dot{\underline{X}} = (\underline{A} + \underline{B}\,\underline{K})\,\underline{X} = \underline{A}_{c}\underline{X} \tag{5}$$

Using the above, it is clear that (4) may be written as

$$\underline{P}\,\underline{A}_{c} + \underline{A}_{c}^{T}\underline{P} + \underline{K}^{T}\underline{R}\,\underline{K} + \underline{Q} = 0 \tag{6}$$

Furthermore it is easy to show that for a given desired set of closed loop eigenspectrum $\Lambda_{d}(\underline{A}_{c})$, where $\Lambda(.)$ denotes the eigenspectrum of the matrix argument, (6) can be written as

$$\underline{M}^{T}\underline{P}\,\underline{M}\,\underline{\Sigma} + \underline{\Sigma}\,\underline{M}^{T}\underline{P}\,\underline{M} = -\underline{M}^{T}(\underline{Q} + \underline{K}^{T}\underline{R}\,\underline{K})\,\underline{M} \tag{7}$$

where $\underline{M}$ is the modal matrix for the closed loop system matrix $\underline{A}_{c}$, and $\underline{\Sigma}$ is a diagonal matrix which has the desired closed loop eigenspectrum[1] on its main diagonal, and is given by

$$\underline{\Sigma} = \underline{M}^{-1}\underline{A}_{c}\,\underline{M} \tag{8}$$

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[1] Throughout this paper it is assumed that the system's eigenvalues are distinct.

The solution $\underline{P}$ to the AMRE can be obtained [4] without a need to solving the AMRE by assuming that matrix $\underline{Q}$ satisfies

$$\underline{Q} = (\underline{M}\,\underline{M}^T)^{-1} - \underline{K}^T\underline{R}\,\underline{K} \tag{9}$$

and is given by

$$\underline{P} = -\frac{1}{2}(\underline{M}\,\underline{\Sigma}\,\underline{M}^T)^{-1} \tag{10}$$

Hence, the optimal feedback gain (3), is given by

$$\underline{K} = \frac{1}{2}\underline{R}^{-1}\underline{B}^T\underline{M}^{-T}\underline{\Sigma}^{-1}\underline{M}^{-1} \tag{11}$$

Clearly, it can be seen from (11) that the optimal control law given in (11) is a function of the modal matrix of the closed loop system, which is in turn a function of the closed loop eigenvector $(\underline{V}_i)$ corresponding to the eigenvalue (λ_i) of the resulting closed loop system. The closed loop eigenstructure is governed by

$$\underline{A}_c\underline{V}_i = \lambda_i\underline{V}_i \qquad \forall\ i = 1,2,\ldots,n \tag{12}$$

Using (5), this equation can be put in the form

$$[(\underline{A} - \lambda_i\underline{I}) \quad \underline{B}]\begin{pmatrix}\underline{V}_i\\ \underline{K}\,\underline{V}_i\end{pmatrix} = 0 \tag{13}$$

In order for the above equality to hold the following is required

$$\begin{pmatrix}\underline{V}_i\\ \underline{K}\,\underline{V}_i\end{pmatrix} \in \aleph_i([(\underline{A} - \lambda_i\underline{I}) \quad \underline{B}]) \qquad \forall\ i = 1,2,\ldots,n \tag{14}$$

where $\aleph(.)$ denotes the Kernel space of the mapping in the argument. It is now easy to show that the feedback gain $\underline{K}$ is given [5] by

$$\underline{K} = \underline{N}\,\underline{M}^{-1} \tag{15}$$

where the mxn matrix $\underline{N}$ is given by

$$\underline{N} = [\underline{K}\,\underline{V}_1 \quad \underline{K}\,\underline{V}_2 \quad \ldots\ \underline{K}\,\underline{V}_n] \tag{16}$$

Equating (15), and (11) will now result in

$$\underline{M}\,\underline{\Sigma}\,\underline{N}^T = -\frac{1}{2}\,\underline{B}\,\underline{R}^{-1} \qquad (17)$$

The optimal feedback law that accomplishes the eigenspectrum assignment is given by equation (11). The unknowns $\underline{M}$, and $\underline{N}$ are determined from the algebraic system of (nm) equations in (nm) unknowns given in (17), in conjunction with the constraint imposed by (14). Finally, the weighting matrix that would results in the optimal modal controller can be obtained from equation (9).

3) Modal Controller for Aggregated System

A computational algorithm for the optimal LQR eigenstructure assignment based on the above approach is sugges ‡ in [5]. However, the suggested algorithm would still involve full order system of equations even if a small subset of the eigenspectrum needs be placed. In the following discussion a sequential approach to eigenstructure placement is presented. It is shown that it is possible to assign a real or complex conjugate pair of eigenspectrum at a time without altering the rest of the eigenspectrum. The advantage offered by this is the computational simplicity that this approach will offer. This is because at each stage of the recursive algorithm the original nth order system is aggregated to a first or a second order system depending on the eigenvalues to be placed. This in turn will result in eigenstructure placement for a scalar, or a second order system which is computationally very simple to perform. Before we state the key theorem which will form the basis of the proposed approach, a few properties of the Hamiltonian matrix for the LQR problem stated in (1), and (2) are pointed out.

It is a property of (4) that its solution $\underline{P}$ satisfies [7]

$$\det(s\underline{I} - \underline{A} + \underline{B}\,\underline{R}^{-1}\underline{B}^T\underline{P}\,)\,\det(s\underline{I} + \underline{A} - \underline{P}\,\underline{B}\,\underline{R}^{-1}\underline{B}^T) = \det(s\underline{I} - \underline{H}\,) \qquad (18)$$

where

$$\underline{H} = \begin{pmatrix} \underline{A} & -\underline{B}\,\underline{R}^{-1}\underline{B}^T \\ -\underline{Q} & -\underline{A}^T \end{pmatrix} \qquad (19)$$

is the 2nx2n Hamiltonian matrix for the optimal LQR problem. It is clear from (18) that the eigenspectrum of the Hamiltonian matrix has a mirror image property, and its stable eigenvalues are the same as those of $\underline{A}_c$ given in (5).

THEOREM - Consider an the following aggregated system

$$\dot{\hat{\underline{X}}} = \hat{\underline{A}}\,\hat{\underline{X}} + \hat{\underline{B}}\,\underline{U} \qquad (20)$$

where $\underline{X}\epsilon R^{\ell}$, with $\hat{\underline{A}}$ and $\hat{\underline{B}}$ appropriately dimensioned, and assume that $\Lambda(\hat{\underline{A}})\subset\Lambda(\underline{A})$. The Hamiltonian system for this system is given by

$$\hat{\underline{H}} = \begin{pmatrix} \hat{\underline{A}} & -\hat{\underline{B}}\,\underline{R}^{-1}\hat{\underline{B}}^T \\ -\hat{\underline{Q}} & -\hat{\underline{A}}^T \end{pmatrix} \qquad (21)$$

For any choice of $\hat{\underline{Q}}$ in the above, it can be proved that if $\underline{Q}$ in (19) is selected as $\underline{Q} = \underline{C}^T\hat{\underline{Q}}\,\underline{C}$, then $\Lambda(\underline{H})\supset\Lambda(\hat{\underline{H}})$, and also the remaining $(2n-2\ell)$ eigenvalues of $\underline{H}$ are given by $\{\,[\Lambda(\underline{A})\cap\Lambda(\hat{\underline{A}})]\cup[-[\Lambda(\underline{A})\cap\Lambda(\hat{\underline{A}})]]\,\}$. Where the aggregation matrix $\underline{C}$ is an ℓxn full rank matrix given by

$$\underline{C} = [\,\underline{I}_{\ell} \quad \underline{0}\,]\,\underline{W}^{-1} \qquad (22)$$

with $\underline{W}$ being the modal matrix of $\underline{A}$ with its first ℓ column being the eigenvectors corresponding to the ℓ eigenvalues that are to be retained in $\hat{\underline{A}}$.

Proof: The eigenvalues of $\underline{H}$ are the roots of the following polynomial

$$\Delta(s) = \det(s\underline{I} - \tilde{\underline{H}}\,)$$

where $\tilde{\underline{H}}$ is obtained as

$$\bar{H} = T^{-1} H T = \begin{pmatrix} \Gamma(A) & -E \\ -W^T Q W & -\Gamma(A) \end{pmatrix}$$

$\Gamma(A)$ is a diagonal matrix whose diagonal elements are eigenvalues of A, and

$$E = W^{-1} B R^{-1} B^T W^{-T} = \left(\begin{array}{c|c} E_1 & E_2 \\ \hline E_2 & E_3 \end{array}\right)$$

$$T = \begin{pmatrix} W & Q \\ Q & -W^{-T} \end{pmatrix}$$

From the above $\Delta(s)$ can be obtained as

$$\Delta(s) = \det(sI - \Gamma(A))\det(sI + \Gamma(A) - W^T Q W (sI - \Gamma(A))^{-1} E)$$

using $Q = C^T \hat{Q} C$, and (22), after partitioning the above we get

$$\Delta(s) = \det\{(sI - \Gamma_1)[(sI + \Gamma_1) - \hat{Q}(sI - \Gamma_1)^{-1} E_1]\}\det[(sI - \Gamma_2)(sI + \Gamma_2)]$$

where $\Gamma_1 = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_\ell)$, and $\Gamma_2 = \text{diag}(\lambda_{\ell+1}, \lambda_{\ell+2}, \ldots, \lambda_n)$, and notice that $\Lambda(\Gamma_1) = \Lambda(\hat{A}) \subset \Lambda(A)$.

Similarly following along the same lines as above it can be shown that the eigenspectrum of $\hat{H}$ are given by the roots of the following polynomial

$$\hat{\Delta}(s) = \det\{(sI - \Gamma_1)[(sI + \Gamma_1) - \hat{Q}(sI - \Gamma_1)^{-1} E_1]\}$$

Finally, substituting $\hat{\Delta}(s)$ into the expression for $\Delta(s)$, will establish the proof.■

It should be pointed out that the requirement of $\Lambda(\hat{A}) \subset \Lambda(A)$, will be satisfied by the following transformation [8] on the system in (1)

$$\hat{A} = C A C^+ = C A C^T (C C^T)^{-1} \qquad (23\text{-a})$$

$$\hat{B} = C B \qquad (23\text{-b})$$

where $\underline{C}^{+}$ is the Moore-Penroze pseudo inverse of $\underline{C}$. Also, when placing a complex conjugate pair $-\alpha \pm j\beta$, in order to deal with real matrices, these transformations are modified to

$$\hat{\underline{A}} = \underline{L}^{-1}\underline{C}\ \underline{A}\ \underline{C}^{+}\underline{L} = \begin{pmatrix} -\alpha & \beta \\ -\beta & \alpha \end{pmatrix} \tag{24-a}$$

$$\hat{\underline{B}} = \underline{L}^{-1}\underline{C}\ \underline{B} \tag{24-b}$$

where $$\underline{L} = \begin{pmatrix} 0.5 & +j0.5 \\ 0.5 & -j0.5 \end{pmatrix}$$

and

$$\underline{Q} = \underline{C}^{T}\underline{L}^{-T}\hat{\underline{Q}}\ \underline{L}^{-1}\underline{C} \tag{25}$$

The above theorem form the basis of the recursive algorithm. At each stage of the algorithm the dynamical system is aggregated to a first, order system if a real pole is to be assigned, or a second order system if it is a complex conjugate pair that is to be placed. The entire eigenstructure assignment approach, and weight selection approach for the LQR discussed in section 2 is then used to assign the one real, or the complex conjugate of poles of the reduced order system. Once the optimal feedback gain ($\hat{\underline{K}}$), and the desired weighting matrix ($\hat{\underline{Q}}$) for this system are found, they are transformed to the original higher dimensional space by the following transformations:

a) For the case of a real pole

$$\underline{K} = \hat{\underline{K}}\ \underline{C} \tag{26-a}$$

$$\underline{Q} = \underline{C}^{T}\hat{\underline{Q}}\ \underline{C} \tag{26-b}$$

b) For a complex conjugate pair

$$\underline{K} = \hat{\underline{K}}\ \underline{L}^{-1}\underline{C} \tag{27-a}$$

$$Q = G^T L^{-T} \hat{Q} L^{-1} G \qquad (27\text{-b})$$

The above weighting matrix, and the optimal feedback gain is the one that would assign the one real or the complex conjugate pair to a desired location without altering the rest of the eigenspectrum of the system. Successive application of the above theorem would accomplish the assignment of all or a subset of the eigenspectrum. The following lemma will establish the missing link in between each of the stages of the recursive process and would indicate how the final weighting matrix, and the optimal controller are obtained.

LEMMA - Assume that $k \leq n$ is the total number of stages involved in the sequential design algorithm. Given that

$$Q = \sum_{i=1}^{k} Q_i \qquad (28)$$

where Q_i is the desired weighting matrix obtained from either (26-b), or (27-b) at stage i, and the following is satisfied

$$P_i A_i + A_i^T P_i - P_i B R^{-1} B^T P_i + Q_i = 0 \qquad (29)$$

where

$$A = A_1 \quad \text{and} \quad A_{i+1} = A_i - B R^{-1} B^T P_i = A_i + B K_i \qquad (30)$$

then the solution P of (4) can be obtained as

$$P = \sum_{i=1}^{k} P_i \qquad (31)$$

and therefore the desired optimal feedback gain in (3) is given by

$$K = -R^{-1} B^T \sum_{i=1}^{k} P_i \qquad (32)$$

proof: the lemma can be simply proved by adding up the equations in (29), and using (30) to obtain

$$\sum_{i=1}^{k} \left(\underline{P}_i \underline{A} + \underline{A}^T \underline{P}_i - \sum_{j=1}^{k} \underline{P}_i \underline{B} \underline{R}^{-1} \underline{B}^T \underline{P}_j + \underline{Q}_i \right) = 0 \qquad (33)$$

now substitution of (28), and (31) in (33) and the fact that the solution to (4) is unique, completes the proof.■

The above lemma completes the discussion of the proposed approach. Equations (28), and (32) give the value for the final desired weighting matrix, and the optimal feedback gain respectively.

4) Illustrative Example

Consider the unstable multi-input system described by

$$\dot{\underline{X}} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & 4 & -1 \end{pmatrix} \underline{X} + \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \underline{U}$$

The eigenspectrum for the open loop system is given by $\Lambda(\underline{A})=\{-2,-1,2\}$. Suppose that it is desired to stabilize the system and that the desired eigenspectrum for the closed loop system is $\Lambda(\underline{A}_c)=\{-2,-4,-3\}$. The assumption is therefore, that the open loop eigenvalue at (-2) is to be retained while to achieve stability, and sufficient degree of exponential stability the other two are to be moved. Additionally, assume that $\underline{R}$ has been selected to be identity. The sequential algorithm then starts at stage 1.

Stage i=1 At this stage it is desired to move the eigenvalue at 2 to -3. The aggregation matrix $\underline{C}$, for $\underline{A}_1 = \underline{A}$ is found from (22) to be

$$\underline{C} = [0.6667 \quad 1.0 \quad 0.3333]$$

and thus the reduced order scalar system is given by

$$\dot{\hat{X}} = 2\hat{X} + [0.6667 \quad 0.3333]\, \underline{U}$$

Using (14), and (17) along with (11) results in the

$$\hat{\underline{K}}_1 = \begin{pmatrix} -6 \\ -3 \end{pmatrix}$$

also from (9)

$$\hat{Q}_1 = 9.0$$

The optimal feedback gain, and the corresponding weighting matrix that accomplishes the pole placement is now obtained from (26) as

$$K_1 = \begin{pmatrix} -4.0 & -6.0 & -2.0 \\ -2.0 & -3.0 & -1.0 \end{pmatrix}$$

$$Q_1 = \begin{pmatrix} 4.0 & 6.0 & 2.0 \\ 6.0 & 9.0 & 3.0 \\ 2.0 & 3.0 & 1.0 \end{pmatrix}$$

Notice now that application of the above optimal feedback gain will result in a closed loop system with poles at (-2,-1,-3), and the system is now stabilized. An additional stage is required however, to place the pole at -1 at a more desirable location. Therefore, A_2 is now be found from (30) and the process continues.

Stage i-2 At this stage A_2 is aggregated and exact similar steps as in stage i-1 are followed to move the pole at -1 to -4. The feedback gain K_2, and the weighting matrix Q_2 were calculated.

Since all the poles are placed, the desired overall feedback gain, and weighting matrix were found from (32), and (28) respectively to be

$$K = \sum_{i=1}^{2} K_i = \begin{pmatrix} -4.9231 & -9.2308 & -3.3846 \\ -3.3846 & -7.8462 & -3.0769 \end{pmatrix}$$

$$Q = \sum_{i=1}^{2} Q_i = \begin{pmatrix} 8.6154 & 22.1538 & 8.9231 \\ 22.1538 & 65.5385 & 27.2308 \\ 8.9231 & 27.2308 & 11.3846 \end{pmatrix}$$

The control law $U = K X$ with K given as above will place the eigenspectrum of the closed loop system at desired locations, and at the same time will minimize the cost functional (2) with Q given as above.

5) Conclusion

A sequential algorithm for optimal LQR weight selection, for

achieving a desired set of predefined eigenspectrum by the optimal controller was presented in this paper. The approach taken to accomplish the pole placement and the weight selection is based on the entire eigenstructure selection scheme of [4], and [5], and does not require solving the AMRE. The advantage offered by the proposed approach is in its computational simplicity. The algorithm becomes computationally attractive by aggregating the system to a first or a second order system, and placing a single real, or a complex conjugate pole at a time. This amounts to working with scalar or second order matrices almost throughout the design. This simplicity is particularly important when dealing with large scale ill conditioned systems whose dominant or unstable modes need to be placed, since high order matrix computations are avoided in such cases. Finally, the algorithm is capable of placing the entire or a subset of eigenspectrum, so the control effort need not be spend to move nondominant poles.

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